# Thue's equation and a conjecture of Siegel

by


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## 1. Introduction

In his fundamental work on diophantine equations $f(x, y)=0$, Siegel [18] remarks that in the case when the curve defined by this equation is irreducible and of positive genus, it is possible to find an explicit bound for the number of integer solutions. He then says "Man kann nun vermuten, dass sich sogar eine Schranke finden lässt, die nur von der Anzahl der Koeffizienten abhängt", i.e. one may conjecture that a bound may be derived which depends only on the number of coefficients.

In this form, the conjecture is not true. Chowla [2] showed that for given $k\neq 0$, the number $N_k(h)$ of solutions of $x^3+ky^3=h$ has $N_k(h)=\Omega_k(\log\log h)$, i.e. there is a number $\gamma_k>0$ such that $N_k(h)>\gamma_k \log\log h$ for infinitely many positive values of $h$. More generally, consider cubic Thue equations, i.e. equations

$$F(x, y) = h, \tag{1.1}$$

where $F$ is a cubic form with integer coefficients which is irreducible over the rationals. According to Mahler [11], the number $N_F(h)$ of solutions of such an equation has $N_F(h)=\Omega_F(\log^{1/4} h)$, and this was improved to $N_F(h)=\Omega_F(\log^{1/3} h)$ by Silverman [19]. In fact, Silverman shows the existence of infinitely many (non-equivalent) cubic forms $F$ as above with $N_F(h)=\Omega_F(\log^{2/3} h)$. On the other hand it is conceivable that the number $N'_F(h)$ of *primitive* solutions (i.e. solutions with coprime $x, y$) of a cubic Thue equation (1.1) is under some absolute bound. Also, it is possible that Siegel's conjecture is true for curves of genus $>1$.


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