# On the classification of $G$-spheres I: equivariant transversality

by

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This paper is the first in a series of three. Stated in geometric terms the papers examine locally linear group actions on spheres for odd order groups $G$. In essentially equivalent homotopy theoretic terms the papers study the homotopy types of the spaces $PL_G(V)$ and $\mathrm{Top}_G(V)$ of equivariant $PL$-homeomorphisms and homeomorphisms of a linear representation $V$. In fact, it is the homogeneous spaces $F_G(V)/PL_G(V)$ and $\mathrm{Top}_G(V)/PL_G(V)$ we study where $F_G(V)$ is the space of proper equivariant homotopy equivalences. Our results generalize theorems of Haefliger, Kirby-Siebenmann, Sullivan and Wall, and others.

The link between classification of manifolds and classification of homotopy types is transversality and this is the subject of the present first paper. The second paper generalizes Wall's classification of fake lens spaces to the classification of $G$-spheres which are equivariantly homotopy equivalent to a given linear action. This involves the determination of the equivariant homotopy type of the $G$-space $F/PL$ and a complete calculation of the $PL$ equivariant surgery sequence for a linear $G$-sphere. As a result we show that the homotopy groups of $PL_G(V\oplus U)/PL_G(V)$ vanish in dimensions less than $\dim V^G$. In the third paper we study $G$ triangulation theory. In particular we study the homotopy groups of $\mathrm{Top}_G(V\oplus U)/\mathrm{Top}_G(V)$ in a range. The homotopy groups of the Stiefel spaces $PL_G(V\oplus U)/PL_G(V)$ and $\mathrm{Top}_G(V\oplus U)/\mathrm{Top}_G(V)$ are in turn needed for equivariant transversality, so the three papers are locked together in an inductive fashion. The reader is referred to § 4 below and to the individual Parts II and III for more information.

We now give a brief discussion of the content of the present paper. Let $G$ denote an arbitrary finite group. We consider the $G$-transversality question in the *locally linear*

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