The existence of surfaces of constant mean curvature with free boundaries

by

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1. Introduction

Suppose S is a surface in $\mathbf{R}^3$ diffeomorphic to the standard sphere S^2 by a smooth diffeomorphism $\Psi: \mathbf{R}^3 \to \mathbf{R}^3$ of class C^4, and let $H \in \mathbf{R}$. In this paper we give a sufficient condition for the existence of an unstable disc-type surface of constant mean curvature H with boundary on S and intersecting S orthogonally along its boundary. In isothermal coordinates such a surface may be parametrized by a map $X \in C^2(B; \mathbf{R}^3) \cap C^1(\bar{B}; \mathbf{R}^3)$ of the unit disc

$$B = B_1(0) = \{(u, v) = w \in \mathbf{R}^2 \mid u^2 + v^2 < 1\}$$

into $\mathbf{R}^3$ satisfying the following conditions:

$$\Delta X = 2H X_u \wedge X_v \tag{1.1}$$

$$|X_u|^2 - |X_v|^2 = 0 = X_u \cdot X_v \tag{1.2}$$

$$X(\partial B) \subset S, \tag{1.3}$$

$$\partial_n X(w) \perp T_{X(w)} S, \quad \forall w \in \partial B. \tag{1.4}$$

Here $X_u = (\partial/\partial u)X$, etc., "$\wedge$" denotes the exterior product in $\mathbf{R}^3$, "$\cdot$" denotes the scalar product, n is the outward unit normal on ∂B, "$\perp$" means orthogonal, and $T_Q S$ denotes the tangent space to S at $Q \in S$.

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