FIBER SPACES OVER TEICHMÜLLER SPACES

BY

LIPMAN BERS

Columbia University, New York, N.Y., USA (1)

To Elisha Netanyahu

Introduction

We first summarize the results of this paper for the simplest and most important special case: the Teichmüller spaces $T(p, n)$ of surfaces of type (p, n), i.e. closed Riemann surfaces of genus p with n punctures. The points of $T(p, n)$ are equivalence classes of orientation preserving homeomorphisms of a fixed surface S_0 of type (p, n) onto other such surfaces; two mappings, f_1 and f_2, are considered equivalent if there is a conformal mapping h such that $f_2^{-1} \circ h \circ f_1$ is homotopic to the identity. Homotopy classes of orientation preserving automorphisms of S_0 form the modular group Mod (p, n) which acts naturally on $T(p, n)$, and $X(p, n) = T(p, n)/\text{Mod}\,(p, n)$ is the space of moduli (conformal equivalence classes) of surfaces of type (p, n). We assume throughout that $3p-3+n \geqslant 0$. The space $T(p, n)$ has a canonical structure of a complex $(3p-3+n)$-dimensional manifold, the action of Mod (p, n) on $T(p, n)$ is holomorphic and properly discontinuous, and $X(p, n)$ is a normal complex space.

A central result in Teichmüller space theory asserts that $T(p, n)$ admits an essentially canonical representation as a bounded domain in $\mathbf{C}^{3p-3+n}$. In proving this result [7] one attaches to every $\tau \in T(p, n)$ a Jordan domain $D(\tau)$ and a quasi-Fuchsian group G^{τ}, both depending holomorphically on τ, such that τ is the equivalence class of mappings of S_0 onto $D(\tau)/G^{\tau}$. The fiber space $F(p, n)$ over $T(p, n)$ is the set of pairs (τ, z), with $\tau \in T(p, n)$, $z \in D(\tau)$.

We shall show that the group Mod (p, n) can be extended to a group mod (p, n) which acts holomorphically and properly discontinuously on $F(p, n)$. The quotient $Y(p, n) = F(p, n)/\text{mod}\,(p, n)$ is a normal complex space and a fiber space over $X(p, n)$ with the

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