The spherical Bernstein problem in even dimensions and related problems

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0. Introduction

The minimal surface equation is probably the best known among the non-linear, elliptic partial differential equations, and has been studied extensively. In Euclidean space $\mathbf{R}^3$ the classical Bernstein theorem states that any solution which is an entire minimal graph over $\mathbf{R}^2$, must be a plane. In a celebrated sequence of investigations the combined efforts of de Giorgi [8], Almgren [1], Simons [16], Bombieri, de Giorgi and Giusti [3] succeeded in extending this result to $\mathbf{R}^n$, $n\leqslant 8$, and providing counterexamples for $n>8$. At the 1970 International Congress of Mathematicians in Nice, Professor S. S. Chern proposed the following as one outstanding problem in differential geometry:

The Spherical Bernstein problem: Let the $(n-1)$-sphere be imbedded as a minimal hypersphere in the standard Euclidean n-sphere $S^n(1)$. Is it necessarily an equator?

For $n=3$ the answer to the above problem was already known to be positive by a theorem of Almgren and Calabi, which holds under the weaker assumption of an immersed S^2 in $S^3(1)$. No further progress was made until Wu-Yi Hsiang recently proved the existence of infinitely many non-congruent minimal imbeddings of S^{n-1} into $S^n(1)$ for the specific dimensions $n=4, 5, 6, 7, 8, 10, 12, 14$ [9, 10].

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