REGULARITY AND SINGULARITY ESTIMATES ON HYPERSURFACES MINIMIZING PARAMETRIC ELLIPTIC VARIATIONAL INTEGRALS

BY

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Introduction

In this paper we study the structure of n dimensional rectifiable currents in $\mathbf{R}^{n+1}$ which minimize the integrals of parametric elliptic integrands. The existence of such minimizing surfaces is well known [7, 5.1.6] as is their regularity almost everywhere [7, 5.3.19]. In Part I of the present paper we give a new geometric construction from which regularity estimates can be obtained for minimizing hypersurfaces. In this construction we replace the parametric problem for n dimensional surfaces in $\mathbf{R}^{n+1}$ by a nonparametric problem for which the minimizing hypersurface is a graph in $\mathbf{R}^{n+2}$ with horizontal slices closely approximating in a certain sense the hypersurface(s) minimizing the original problem. Analysis of the associated Euler-Lagrange partial differential equation carried out in § 2 of Part I yields an upper bound for the integral of the square of the second fundamental form over the approximating graphs, hence over the regular parts of the original surface. Since a neighbourhood of a singular point must contribute substantially to this integral (see Theorem 1.3 and the remark following it), we can thus conclude by an argument similar to that given by Miranda [13] that the Hausdorff $(n-2)$-dimensional measure of the interior singular set is locally finite (Theorem 3.1).

In Part II of this work we show that the singular sets in question must have Hausdorff

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