ASYMPTOTIC BEHAVIOUR OF EIGEN FUNCTIONS ON A SEMISIMPLE LIE GROUP: THE DISCRETE SPECTRUM

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1. Introduction

Let G be a connected noncompact real form of a simply connected complex semisimple Lie group. For many questions of Fourier Analysis on G it is useful to have a good knowledge of the behaviour, at infinity on G, of the matrix coefficients of the irreducible unitary representations of G. In this paper we restrict ourselves to the discrete series of representations of G, and study the rapidity with which the corresponding matrix coefficients decay at infinity on the group.

Let K be a maximal compact subgroup of G. Given any p, with $1 \leqslant p \leqslant 2$, we denote by $\mathcal{E}_p(G)$ the set of all equivalence classes of irreducible unitary representations of G whose K-finite matrix coefficients are in $L^p(G)$; $\mathcal{E}_2(G)$ is then the discrete series of G, while $\mathcal{E}_{p'}(G) \subseteq \mathcal{E}_p(G)$ for $1 \leqslant p' \leqslant p \leqslant 2$. We assume that $\operatorname{rk}(G) = \operatorname{rk}(K)$ so that $\mathcal{E}_2(G)$ is nonempty. Let Ξ and σ be the spherical functions on G defined in [15]. Then it follows from the work in [14] that, if $\omega \in \mathcal{E}_2(G)$ and if f is a K-finite matrix coefficient of (a representation belonging to) ω, one can find constants $c > 0$, $\gamma > 0$, $q \geqslant 0$ (depending on f) such that

$$|f(x)| \leqslant c\Xi(x)^{1+\gamma}(1+\sigma(x))^q \quad (x \in G). \tag{1.1}$$

Given $\omega \in \mathcal{E}_2(G)$ and a number $\gamma > 0$, we shall say that ω is *of type* γ if the K-finite matrix coefficients of ω satisfy (1.1) for suitable $c > 0$, $q \geqslant 0$. For a fixed $\omega \in \mathcal{E}_2(G)$ it is then natural to ask what is the largest $\gamma > 0$ for which ω is of type γ. In particular, it is natural to ask for necessary and sufficient conditions in order that $\omega \in \mathcal{E}_p(G)$ $(1 \leqslant p < 2)$.

Let $\mathfrak{g}$ be the Lie algebra of G, and $\mathfrak{g}_c \supseteq \mathfrak{g}$ the complexification of $\mathfrak{g}$. Let $B \subseteq K$ be a Cartan subgroup of G; $\mathfrak{b}$, the Lie algebra of B; and $\mathfrak{b}_c = \mathbf{C}\cdot\mathfrak{b}$. Let $\mathcal{L}_{\mathfrak{b}}$ be the additive group of all

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