THE EXTERIOR NONSTATIONARY PROBLEM FOR THE NAVIER–STOKES EQUATIONS

BY

JOHN G. HEYWOOD

University of British Columbia

I. Introduction

Although the exterior stationary problem for the Navier–Stokes equations has been proved by Leray [1] to possess a solution under very general circumstances, it is unknown even in the case of small data whether Leray's solution of the problem is unique or whether it may be formed as the limit of a nonstationary solution as $t \to \infty$. In this paper we prove that for a particular class of prescribed boundary values there is exactly one stationary solution attainable as the limit, starting from rest, of a physically reasonable nonstationary solution. Our method is based on a global existence theorem for the initial boundary value problem which we prove under hypotheses that allow time dependent boundary values and a time dependent velocity at infinity. This theorem assures the unique solvability of the initial boundary value problem whenever there is an approximate solution which is sufficiently good and satisfies a stability condition. This existence theorem has also enabled us to state simple conditions sufficient to ensure the stability of nonstationary solutions of the Navier–Stokes equations defined in arbitrary three-dimensional regions.

The Navier–Stokes equations govern fluid motion in the theory of viscous incompressible flow. The exterior stationary problem for the Navier–Stokes equations consists of finding, in the region exterior to a closed bounded surface, time independent velocity and pressure functions which together solve the equations and are such that the velocity function assumes given values on the surface and tends to a prescribed limit at infinity. Of course, stationary flow occurs in nature only as the limit of nonstationary flow. Presumably solutions of the exterior stationary problem model fluid flows which may be obtained by performing the following idealized experiment with the right choice of prescribed data. An object is immersed in a fluid which occupies all three-dimensional