HALF-ORDER DIFFERENTIALS ON RIEMANN SURFACES

BY

N. S. HAWLEY and M. SCHIFFER

Stanford University, Stanford, Calif., U.S.A. (1)

Dedicated to Professor R. Nevanlinna on the occasion of his 70th birthday

Introduction

In this paper we wish to exhibit the utility of differentials of half integer order in the theory of Riemann surfaces. We have found that differentials of order $\frac{1}{2}$ and order $-\frac{1}{2}$ have been involved implicitly in numerous earlier investigations, e.g., Poincaré's work on Fuchsian functions and differential equations on Riemann surfaces. But the explicit recognition of these differentials as entities to be studied for their own worth seems to be new. We believe that such a study will have a considerable unifying effect on various aspects of the theory of Riemann surfaces, and we wish to show, by means of examples and applications, how some parts of this theory are clarified and brought together through investigating these half-order differentials.

A strong underlying reason for dealing with half-order differentials comes from the general technique of contour integration; already introduced by Riemann. In the standard theory one integrates a differential (linear) against an Abelian integral (additive function) and uses period relations and the residue theorem to arrive at identities. As we shall demonstrate, one can do an analogous thing by multiplying two differentials of order $\frac{1}{2}$ and using the same techniques of contour integration.

As often happens, when one discovers a new (at least to him) entity and starts looking around to see where it occurs naturally, one is stunned to find so many of its hiding places —and all so near the surface.

Our current point of view concerning the study of Riemann surfaces has evolved from an earlier one in which we introduced the notion of a *meromorphic connection* in analogy with classical notions in real differential geometry; we now view the theory of connections

(1) This work was supported in part by National Science Foundation grant GP 4069 and Air Force contract AF 49(638) 1345 at Stanford University.