UNIFORM APPROXIMATION ON SMOOTH CURVES

BY

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Let $K_1, ..., K_n$ be compact subsets of complex N-space C^N, each the locus of a smooth (*continuously differentiable*) curve. Let $K = K_1 \cup ... \cup K_n$.

For any compact set Y in C^N define its polynomial convex hull $\hat{Y}$ as

$$\{p \in C^N : |f(p)| \leqslant \max_Y |f| \text{ for all polynomials } f\},$$

and say that Y is *polynomially convex* whenever $Y = \hat{Y}$.

Let X be a polynomially convex set in C^N.

THEOREM.

A. $\widehat{K \cup X} - (K \cup X)$ *is a (possibly empty) one-dimensional analytic subset of* $C^N - (K \cup X)$.

B. *Every continuous function on* $K \cup X$ *which is uniformly approximable on* X *by polynomials is uniformly approximable on* $K \cup X$ *by rational functions.*

C. *If* K *is simply-connected and disjoint from* X *or, more generally, if the map* $\check{H}^1(K \cup X; Z) \to \check{H}^1(X; Z)$ *induced by* $X \subset K \cup X$ *is injective then* $K \cup X$ *is polynomially convex.*

Comments (Technical)

1. N may be infinite, but n is finite.

2. A closed subset V of an open subset U of C^N is a *one-dimensional analytic subset of* U if and only if a neighborhood of each point in V can be covered by finitely many sets of the form $\Phi(\Delta)$ where Δ is an open disk in the plane and each $\Phi : \Delta \to V$ is a non-constant *analytic* mapping, i.e., for each complex coordinate z_j on C^N, $z_j \circ \Phi$ is analytic on Δ.

3. $\hat{Y}$ is the spectrum of the algebra of all uniform limits of polynomials on Y [18].

4. In part B, if $K \cup X$ is polynomially convex then the rational functions may be taken to be polynomials [18].

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