CURVES ON 2-MANIFOLDS AND ISOTOPIES

BY

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In [2, 3] R. Baer proved that two homotopic simple closed curves, on an orientable closed 2-manifold M of genus greater than one, are isotopic. He applied this theorem to show that a homeomorphism of M, which is homotopic to the identity, is isotopic to the identity. There have been investigations of the same theorems by W. Brodel [4] and W. Mangler [8].

This paper is a detailed and exhaustive investigation into the theorems of Baer. We shall assume throughout that M is a triangulated connected 2-manifold. But we shall not require M to be compact, nor to be without boundary. The paper is as self-contained as possible. In particular it does not depend on the work cited above.

An essentially new feature is that we treat the case where a basepoint is held fixed. In Theorem 6.3 it is proved that if $h \simeq 1 : M, * \to M, *$, where M is a closed 2-manifold and h is a homeomorphism, then h is isotopic to the identity, keeping the basepoint fixed. This can be regarded as a first step towards proving the conjecture that the space of all such homeomorphisms is contractible, provided M is not a 2-sphere or a projective plane.

In the main part of the paper, we deal with the case where the maps are piecewise linear. In the Appendix we show how to deduce the corresponding topological theorems, by proving that topological imbeddings or homeomorphisms can be approximated by piecewise linear imbeddings or homeomorphisms. In § 5 we prove a number of results for combinatorial manifolds of arbitrary dimension, all of which are more or less already known, but appear not to be in print.

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