Characterizations of almost surely continuous p-stable random Fourier series and strongly stationary processes

by

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Introduction

In 1973, X. Fernique showed that Dudley's "metric entropy" sufficient condition for the a.s. continuity of sample paths of Gaussian processes, is also necessary when the processes are stationary ([6], [7], [10]). In this paper we extend the Dudley–Fernique theorem to strongly stationary p-stable processes, $1<p\leq 2$.

Let G be a locally compact Abelian group with dual group Γ. We say that a real (resp. complex) random process $(X(t))_{t\in G}$ is a *strongly stationary p-stable process*, $0<p\leq 2$, if there exists a finite positive Radon measure m on Γ such that for all $t_1, \ldots, t_n \in G$ and real (resp. complex) numbers $\alpha_1, \ldots, \alpha_n$ we have

$$E\exp i\operatorname{Re}\sum_{j=1}^{n}\bar{\alpha}_j X(t_j)=\exp-\int_\Gamma\left|\sum_{j=1}^{n}\bar{\alpha}_j\gamma(t_j)\right|^p dm(\gamma).$$

We associate with $(X(t))_{t\in G}$ a pseudo-metric d_X on G defined by

$$d_X(s,t)=\left(\int_\Gamma|\gamma(s)-\gamma(t)|^p\,m(d\gamma)\right)^{1/p},\quad \forall s,\ t\in G. \tag{0.1}$$

Let K be a fixed compact neighborhood of the unit element of G. Let $N(K, d_X; \varepsilon)$ denote the smallest number of open balls of radius ε, in the pseudo-metric d_X, which cover K. We will always assume that K is metrizable. We can now state our main result.

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