Forced vibrations of superquadratic Hamiltonian systems

by

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1. Introduction and main result

This paper is concerned with the existence of T-periodic solutions ($T\in\mathbf{R}$, $T>0$) of the following Hamiltonian system

$$\dot{z} = \mathfrak{g}H'(z)+f(t). \tag{1.1}$$

Here,

$$\mathfrak{g} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$$

is the standard skewsymmetric matrix, $z=z(t)=(p,q)\colon \mathbf{R}\to\mathbf{R}^{2N}$, $\dot{z}=dz/dt$, $H\colon \mathbf{R}^{2N}\to\mathbf{R}$ is a given Hamiltonian and $f\colon \mathbf{R}\to\mathbf{R}^{2N}$ is a given function which is assumed to be T-periodic. The function $f(t)$ represents a forcing term and thus periodic solutions of (1.1) are called *forced vibrations* of the system. Here, H will be required to satisfy the following hypotheses.

(H 1) $H\in C^2(\mathbf{R}^{2N},\mathbf{R})$

(H 2) $0<H(z)\leq \theta H'(z)\cdot z, \quad \forall z\in\mathbf{R}^{2N}, \quad |z|\geq R, \quad 0<\theta<\frac{1}{2}$

(H 3) $a|z|^{p+1}-b\leq H(z)\leq a'|z|^{q+1}+b'$ with $1<p\leq q<2p+1$,

where $a, a'>0$, $b, b'\geq 0$ and $R>0$ are constants. $H'(z)\cdot z$ denotes the scalar product in $\mathbf{R}^{2N}$. Condition (H 2) is a usual way to express that the Hamiltonian is superquadratic

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