FOURIER INTEGRAL OPERATORS. II

BY

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Preface

The purpose of this paper is to give applications of the operator theory developed in the first part (*Acta Math.*, 127 (1971), 79–183). These concern the existence and regularity of solutions of a pseudo-differential equation

$$Pu = f \tag{1}$$

in a manifold X. In particular we construct and study parametrices for P.

In the first chapter, Chapter V, we have collected some general facts concerning the calculus of Fourier integral operators needed later. In Chapter VI we then consider the equation (1) under the assumption that P has a principal symbol p which is homogeneous of degree m and real. First we study the propagation of singularities of solutions of (1). If no bicharacteristic curve of P is contained in a compact set in X this leads to semi-global existence theorems and we can then give necessary and sufficient conditions for the operator P to map $\mathscr{D}'(X)/C^\infty(X)$ onto itself. Globally as well as locally these hypotheses are weaker than those made in Hörmander [17, Chap. VIII]. Under the same hypotheses we construct (twosided) parametrices for P, that is, inverses mod C^∞. If the characteristic set is split in a disjoint union of open and closed subsets N^+ and N^- there is mod C^∞ a unique parametrix $E = E(N^+)$ such that for $f \in \mathscr{E}'(X)$ the wave front set of Ef in addition to that of f only contains forward (backward) bicharacteristic half strips emanating from points in N^+ (resp. N^-). When the characteristic set has k components there are 2^k such parametrices. For the Klein-Gordon equation $\Box + m^2$ these are given by the advanced and retarded fundamental solutions, the Feynman "propagator" and its complex conjugate. The difference between the Feynman propagator and the advanced or retarded fundamental

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