ANALYTIC CONTINUATION ACROSS A LINEAR BOUNDARY

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Introduction

To begin with we recall the following classical theorem concerning analytic continuation across a linear segment:

Let $Q = Q_{a,b}$ denote the rectangle $\{x+iy;\ |x|<a,\ |y|<b\}$ and let $Q^{\pm}$ be its intersection with the open upper and lower halfplane respectively. Two functions $f^{\pm}$ holomorphic in $Q^{\pm}$ are analytic continuations of each other across $(-a, a)$ if they have continuous and identical boundary values on $(-a, a)$.

Although the stated conditions are both necessary and sufficient the theorem is nevertheless inadequate in most nontrivial situations, the reason being that the two functions involved usually appear in a form which does not a priori imply either continuity or boundedness at any point on the common boundary. In most cases the a priori knowledge of $f^{\pm}$ consists of a growth limitation at $(-a, a)$ of the form

$$|f^{\pm}(x+iy)| \leqslant e^{h(|y|)}, \tag{1}$$

where $h(t)$ is a given function increasing steadily to ∞ as t tends to 0. The analytic continuation problem for functions satisfying (1) will be divided into two parts, referred to as the convergence problem which is closely related to a theorem by Runge, and the problem of mollification, to be treated in Chapter I and II respectively. The solutions of both are imperative for the formation of a general theory and both have solutions if and only if

$$\int_0^{\delta} \log h(y)\, dy < \infty . \tag{2}$$

If $h(t)$ increases sufficiently slowly to ∞, or more explicitly, if (1) is replaced by

$$|f^{\pm}(x+iy)| = O(|y|^{-k}), \tag{3}$$