A THEOREM ON NEVANLINNA DEFICIENCIES

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We shall prove the following

THEOREM. *Let $f(z)$ be meromorphic and of finite lower order μ in the finite plane, and let $a_1, a_2, \ldots$ be its set of Nevanlinna deficient values. Then*

$$\sum_{\nu} \delta^{\frac{1}{3}}(a_\nu, f) < \infty. \tag{1}$$

This problem seems to have first been considered in 1939 by O. Teichmüller [16; p. 167] who suggested that, in addition to the classical Nevanlinna defect relation

$$\sum_{\nu} \delta(a_\nu, f) \leqslant 2,$$

certain conditions including finite order might imply

$$\sum_{\nu} \delta^{\frac{1}{2}}(a_\nu, f) < \infty. \tag{2}$$

In 1957 W. Fuchs [5] established (2) under only the assumption that $f(z)$ be of finite lower order. This work was subsequently refined by V. Petrenko [13], and I. Ostrovskii and I. Kazakova [9] who concentrated primarily on the bounds for the sum (2); an alternative proof of Fuchs's theorem was given in 1965 by A. Edrei [2; p. 85].

A major advance was made by W. Hayman [8; p. 90] who proved that if $f(z)$ has finite lower order then

$$\sum_{\nu} \delta^{\frac{1}{3}+\varepsilon}(a_\nu, f) < \infty$$

for every $\varepsilon > 0$.

Following Hayman's approach, Petrenko [14], in 1966, proved the convergence of $\sum \delta^{\frac{1}{3}}(a_\nu, f)\,(\log e/\delta(a_\nu, f))^{-1}$ and in the following year E. Bombieri and P. Ragnedda [1] proved the convergence of $\sum (\delta(a_\nu, f)\,\sigma(\delta(a_\nu, f)))^{\frac{1}{3}}$ for suitable functions $\sigma(t)$ satisfying $\int_0 \sigma(t)/t\,dt < \infty$.

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