# REAL HYPERSURFACES IN COMPLEX MANIFOLDS

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## Introduction

Whether one studies the geometry or analysis in the complex number space $\mathbf{C}_{n+1}$, or more generally, in a complex manifold, one will have to deal with domains. Their boundaries are real hypersurfaces of real codimension one. In 1907, Poincaré showed by a heuristic argument that a real hypersurface in $\mathbf{C}_2$ has local invariants under biholomorphic transformations [6]. He also recognized the importance of the special unitary group which acts on the real hyperquadrics (cf. § 1). Following a remark by B. Segre, Elie Cartan took, up again the problem. In two profound papers [1], he gave, among other results, a complete solution of the equivalence problem, that is, the problem of finding a complete system of analytic invariants for two real analytic real hypersurfaces in $\mathbf{C}_2$ to be locally equivalent under biholomorphic transformations.

Let $z^1, ..., z^{n+1}$ be the coordinates in $\mathbf{C}_{n+1}$. We study a real hypersurface $M$ at the origin 0 defined by the equation

$$r(z^1, ..., z^{n+1}, \bar{z}^1, ..., \bar{z}^{n+1}) = 0, \tag{0.1}$$

where $r$ is a real analytic function vanishing at 0 such that not all its first partial derivatives are zero at 0. We set

$$z = (z^1, ..., z^n), \quad z^{n+1} = w = u + iv. \tag{0.2}$$

After an appropriate linear coordinate change the equation of $M$ can be written as

$$v = F(z, \bar{z}, u), \tag{0.3}$$

where $F$ is real analytic and vanishes with its first partial derivatives at 0. Our basic assumption on $M$ is that it be nondegenerate, that is, the Levi form

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