FUNDAMENTAL SOLUTIONS FOR DEGENERATE PARABOLIC EQUATIONS

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Introduction

Consider a system of n stochastic differential equations

$$d\xi(t) = b(\xi(t))\,dt + \sigma(\xi(t))\,dw(t) \tag{0.1}$$

where $b=(b_1, ..., b_n)$, $\sigma=(\sigma_{ij})$ is an $n\times n$ matrix and $w=(w^1, ..., w^n)$ is n-dimensional Brownian motion. Under standard smoothness and growth conditions on b and σ, the process $\xi(t)$ is a diffusion process (see [7], [8], [11]) with the differential generator

$$L = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i(x)\frac{\partial}{\partial x_i},$$

where $a_{ij}=\frac{1}{2}\sum_k \sigma_{ik}\sigma_{jk}$. Denote by $q(x, t, A)$ the transition probabilities of the diffusion process. If L is elliptic then a fundamental solution for the Cauchy problem associated with the parabolic equation

$$Lu - \frac{\partial u}{\partial t} = 0 \tag{0.2}$$

can be constructed, under suitable smoothness and growth conditions on the coefficients (see [3], [1]); denote it by $K(x, t, \xi)$. It is also known (see [7], [8]) that this fundamental solution is the density function for the transition probabilities of (0.1), i.e.,

$$q(t, x, A) = \int_A K(x, t, \zeta)\,d\zeta \tag{0.3}$$

for any $t>0$, $x\in R^n$, and for any Borel set A in R^n.

The present work is concerned with the case where L is degenerate elliptic, i.e., the

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