CHARACTERS OF CONNECTED LIE GROUPS[1]

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Introduction

Let G be a finite group and $\mathfrak{G}$ the group algebra of G over the complex field. If U is a unitary representation of G on a finite dimensional unitary space H, U extends to a unique * homomorphism U' of $\mathfrak{G}$ into the full operator algebra of H, and conversely. U is multiple of an irreducible representation (or U is a factor representation) if and only if the kernel of U' is a (two-sided) prime ideal of $\mathfrak{G}$. If U and V give rise to the same prime ideal, they are multiples of the same irreducible representation, in which case we call them quasi-equivalent. We denote by Prim (G) the set of all prime ideals of $\mathfrak{G}$ and by $\hat{G}$ the family of all quasi-equivalence classes of factor representations of G. Summing up, there is a canonical bijection between any two of the following three sets: Prim (G), $\hat{G}$ and the set of all characters of G.

Let now G be a separable locally compact group. The theory of characters of such groups was initiated by R. Godement (cf. [11], [12], [13]). One major outgrowth of his investigations was the recognition of the fact that, in order that one should be able to associate with a (in general now infinite dimensional) continuous unitary representation of G a character, beside generating a factor in the sense of F. J. Murray and J. v. Neumann, it must carry a special property, to be called normalcy in the sequel. In particular for this, in general, irreducibility is neither necessary nor sufficient. The notion of character inspired by Godement's work was formalized in the language of C^* algebras by A. Guichardet (cf. [14], and [4], § 17, p. 305). We recall (cf. [4], 13.9, p. 270), that we can associate with G a C^* algebra $C^*(G)$ (to be denoted in the following by $\mathfrak{G}$) such that there is a canonical bijection between continuous unitary representations of G and nondegenerate * representations of $\mathfrak{G}$. This being so, we call a unitary representation U normal, if (1) The ring of operators (v. Neumann algebra) $\mathbf{M}$ generated by U is a semifinite factor, (2) If Φ

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