ABSOLUTELY SUMMING OPERATORS AND LOCAL UNCONDITIONAL STRUCTURES

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1. Introduction

In his remarkable paper [8] Grothendieck defined a one absolutely summing operator between two Banach spaces, to be an operator which maps every unconditionally convergent series to an absolutely convergent series (see definition below). It is well known that a one absolutely summing operator factors through an $L_\infty(\mu)$-space and for every $p\,(1 \leqslant p < \infty)$ also through a certain subspace of $L_p(\mu)$. It was asked in [8] problem 2, p. 72 whether every one absolutely summing operator can be factored through an $L_1(\mu)$-space, and other equivalent formulations of the problem were presented. We establish here the negative answer to this question and related results as well.

The literature on one absolutely summing maps, and more generally p-absolutely summing maps introduced by Pietsch [22], is very extensive and varied. Some results of Grothendieck are by now classical, such as the facts that every operator from an $L_1(\mu)$-space to a Hilbert space is one absolutely summing, and every operator from $L_\infty(\mu)$ to $L_1(\mu)$ is 2-absolutely summing [8], [18]. However, we shall generally make use here only of the definitions and basic results on these spaces. The class of p-absolutely summing operators forms only a single example in the classes of Banach ideals of operators. Equally important, and related by duality, are the Banach ideals of p-integral operators, and L_p-factorizable operators which we mention later in this section.

Our approach to the problem mentioned is to consider various inclusion maps I_n: $E_n \to F_n$ $(n=1, 2, \ldots)$ between certain sequences of finite-dimensional Banach spaces and carefully evaluate the ratios $\gamma_1(I_n)/\pi_1(I_n)$ between their L_1-factorizable norms and

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