THE INHOMOGENEOUS MINIMA OF BINARY QUADRATIC FORMS (II).

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Introduction.

This paper continues the work of our previous paper[1], with which we shall assume that the reader is familiar. There we were mainly concerned with finding the inhomogeneous minimum $M(f)$ of a rational indefinite binary quadratic form; and occasionally the methods yielded also the second minimum $M_2(f)$. We show here that our methods may be extended to deal with the problem of finding an enumerably infinite sequence of minima.

This problem has been solved for the particular forms $x^2 + xy - y^2$ and $x^2 - 2y^2$, by Davenport [1] and Varnavides [2] respectively. The method used by these authors is synthetic, and has the disadvantage of giving no information on the values of $M(f, P)$ close to (but less than) the limiting value $M' = \overline{\lim}\, M(f, P)$.

We consider in sections 1 and 2 the norm-forms $x^2 - 11y^2$ and $x^2 + xy - 3y^2$. The first of these was chosen as the simplest form whose second minimum was not easily established by the methods of our previous paper. We obtain for it, in Theorem 1, a result precisely analogous to those found by Davenport and Varnavides, with an additional clause on the existence of a non-enumerable infinity of incongruent points P with $M(f, P) > M' - \varepsilon$; and we may regard this as an entirely typical result.

In Theorem 7 of our previous paper, we proved that for the form $x^2 + xy - 3y^2$ the first minimum $M(f)$ is taken at both rational and irrational points; its behaviour might therefore be expected not to conform to the usual pattern. We show in fact,

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[1] "*The inhomogeneous minima of binary quadratic forms* (I)", Acta Math. Vol. 87, 1952. References to literal theorems and to the bibliography refer to this paper.