THE CONGRUENCE $ax^3+by^3+c \equiv 0 \pmod{xy}$, AND INTEGER SOLUTIONS OF CUBIC EQUATIONS IN THREE VARIABLES.

By

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I have recently[1] proposed the following

Conjecture: *Let $f(x, y, z)$ be a cubic polynomial in x, y, z with integer coefficients such that $f(x, y, z) - a$ is irreducible for all a. Then if the equation*

$$f(x, y, z) = 0 \tag{1}$$

does not represent a cone in three dimensional space and has one solution in integers, there exists an infinity of integer solutions.

This conjecture, as far as I know, has not been proved for even simple equations such as

$$x^3 + y^3 + z^3 = 3,$$

but was proved for some equations and in particular for

$$z^2 - k^2 = lx + my + Ax^3 + Bx^2y + Cxy^2 + Dy^3,$$

where the coefficients are integers and l is prime to m, the known solution being $x = 0$, $y = 0$, $z = k$. The case $l = m = 0$ seems more difficult, but interesting results can be found for some equations of the form

$$z^2 - k^2 = Ax^3 + By^3. \tag{2}$$

I find that integer solutions of (2) can be deduced from the integer solutions of some very simple equations included in (1), namely,

$$ax^3 + by^3 + c = xyz, \tag{3}$$

[1] "On cubic equations $z^2 = f(x, y)$ with an infinity of integer solutions" *Proceedings of the American Mathematical Society* 3 (1952), 210—217.