

STATE SPACES OF C^* -ALGEBRAS

BY

ERIK M. ALFSEN, HARALD HANCHE-OLSEN and
FREDERIC W. SHULTZ

University of Oslo, Norway and Wellesley College, Wellesley Mass., U.S.A.

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§ 1. Introduction

The purpose of this paper is to characterize the state spaces of C^* -algebras among the state spaces of all JB -algebras. In a previous paper [6] we have characterized the state spaces of JB -algebras among all compact convex sets. Together, these two papers give a complete geometric characterization of the state spaces of C^* -algebras.

Recall from [6] that the state spaces of JB -algebras will enjoy the *Hilbert ball property*, by which the face $B(\varrho, \sigma)$ generated by an arbitrary pair ϱ, σ of extreme states is (affinely isomorphic to) the unit ball of some real Hilbert space, and that there actually exist such faces of any given (finite or infinite) dimension for suitably chosen JB -algebras. In the present paper we show that for an arbitrary pair ϱ, σ of extreme states of a C^* -algebra, then the dimension of $B(\varrho, \sigma)$ is three or one. This statement, which we term the *3-ball property*, is the first of our axioms for state spaces of C^* -algebras. The second and last axiom is a requirement of orientability: the state space K of a JB -algebra with the 3-ball property is said to be *orientable* if it is possible to make a “consistent” choice of orientations for the 3-balls $B(\varrho, \sigma)$ in the w^* -compact convex set K , the idea being that the orienta-