THE IDEAL CENTER OF PARTIALLY ORDERED VECTOR SPACES

BY

WILBERT WILS

Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

Introduction and summary

This paper is concerned with a partially ordered vector space E over R such that $E=E^+-E^+$. The *ideal center* Z_E of E is the algebra of endomorphisms of E which are bounded by a multiple of the identity operator I. Z_E turns out to be a very useful tool in digging up remnants of lattice structure. It provides e.g. a missing link between the theory of simplicial spaces introduced by Effros [13] and the theory of C^*-algebras. As a result a simple proof is obtained of the general extension theorem [1, 5.2.] for certain functions on the extreme boundary of compact convex sets in locally convex spaces proved by Andersen and Alfsen. A unified treatment is given of maximal measures on simplices and central measures on state spaces of C^* algebras.

In § 1 the algebraic foundations for the subsequent theory are laid.

If the ordering on E is Archimedean then Z_E is isomorphic to a dense subalgebra of $C(\Omega)$ where Ω is a compact Hausdorff space. The relation of Z_E with relics of lattice structure becomes clear in studying the idempotents in Z_E which are precisely the extremal points of $(Z_E)_1^+$. The images SE^+ of such elements are called split-faces of E^+ because they induce a splitting of E, which is similar to the decomposition in disjoint complementary bands in the lattice setting. An important property of the set of split-faces is that it is a Boolean algebra. The concept of split-faces can be "localized" by considering spaces $F-F$, with F a face of E^+ and split-faces of F within $F-F$. This gives rise to a *disjointness* relation, $\circ\!\!\!\downarrow$, for faces and elements of E^+. A geometric characterization of disjointness for two faces F, G is that if $0\leqslant k\leqslant f+g$ with $f\in F$, $g\in G$ then k admits a unique decomposition $k=k_1+k_2$ with $0\leqslant k_1\leqslant f$; $0\leqslant k_2\leqslant g$. Then k_1 is the infimum in E of k and f. These notions and propositions can be generalized to more than two of course.

A map R from one partially ordered vector space into another is said to be *bipositive* if R is positive and $Rk\geqslant 0$ implies $k\geqslant 0$. If $k\in E^+$ and $J_k=\{T\in Z_E \mid Tk=0\}$ then J_k is a