# THE LIMIT SET OF A FUCHSIAN GROUP

BY

S. J. PATTERSON

*University of Cambridge, England*

## § 1. Introduction

In this paper we shall consider the limit set of a finitely generated Fuchsian group of the second kind. In particular, we shall attempt to calculate the Hausdorff dimension of the limit set. If the group, $G$, has no parabolic elements this is actually achieved in section 4, whereas, if $G$ has parabolic elements we can obtain partial results which are discussed in section 5. The proof of these results involves the construction of a measure supported on the limit set, from which, at least in principle, we can obtain a lower bound for the Hausdorff dimension. This bound is the same as the upper bound found by Beardon [4].

The actual measure which we construct proves to be intimately related to the theory of the Laplace operator on $G\backslash\mathbf{H}$, and consequently we obtain new insights into both of these. This allows us, for example, to give a new proof of a theorem of Beardon [4] (see the Corollary to Theorem 7.1).

Section 2 recalls some well-known, but difficult to locate, results on the description of the geometry of Fuchsian groups of the second kind. These are used in making various estimates.

By way of notation, we shall take all Fuchsian groups to be finitely generated, although several of the results will be valid without this restriction (in particular those of section 3). These groups will be assumed to act on the unit disc $\Delta$ unless stated otherwise. For a domain $D$ we shall write Con $(D)$ for the group of conformal homeomorphisms of $D$ onto $D$. We can, as usual, represent an element $g \in$ Con $(\Delta)$ as a matrix

$$\begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \quad (\alpha, \beta \in \mathbf{C}, |\alpha|^2 - |\beta|^2 = 1),$$

and if this is so we shall write

$$\mu(g) = 2(|\alpha|^2 + |\beta|^2).$$