ON THE NON-LINEAR COHOMOLOGY OF LIE EQUATIONS. II

BY

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CHAPTER II. NON-LINEAR COHOMOLOGY

7. Lie equations and their non-linear cohomology

Let $R_k \subset J_k(T)$ be a differential equation; set $R_{k-1} = J_{k-1}(T)$, $R_{k-2} = J_{k-2}(T)$, $\tilde{R}_{k+l} = \nu^{-1} R_{k+l} \subset \tilde{J}_{k+l}(T)$, $R^0_{k+l} = R_{k+l} \cap J^0_{k+l}(T)$, $\tilde{\mathcal{R}}_{k+l} = \nu^{-1} \mathcal{R}_{k+l} \subset \tilde{J}_{k+l}(\mathcal{T})$, and set $\tilde{J}_l(R_k) = \nu^{-1} J_l(R_k) \subset \tilde{J}_{(l,k)}(T)$. For $l \geqslant -1$, let $g_{k+l} \subset S^{k+l} J_0(T)^* \otimes J_0(T)$ be the kernel of π_{k+l-1}: $R_{k+l} \to R_{k+l-1}$ or of π_{k+l-1}: $\tilde{R}_{k+l} \to \tilde{R}_{k+l-1}$.

Definition 7.1. A differential equation $R_k \subset J_k(T)$ is a Lie equation if $[\tilde{\mathcal{R}}_k, \tilde{\mathcal{R}}_k] \subset \tilde{\mathcal{R}}_k$.

It follows from (1.15) and (1.16) that

$$[\tilde{\mathcal{R}}_{k+1}, \mathcal{R}_k] \subset \mathcal{R}_k \text{ and } [\mathcal{R}_{k+1}, \mathcal{R}_{k+1}] \subset \mathcal{R}_k. \tag{7.1}$$

On the other hand, we have, for all $l \geqslant 0$,

$$[\tilde{\mathcal{R}}_{k+l}, \tilde{\mathcal{R}}_{k+l}] \subset \tilde{\mathcal{R}}_{k+l} \tag{7.2}$$

(cf. Proposition 4.3 of [19]). In particular, if R_{k+l} is a vector bundle, then R_{k+l} is a Lie equation and

$$\tilde{R}_{k+l} = \tilde{\lambda}_l^{-1}(\tilde{J}_l(R_k)) \tag{7.3}$$

where $\tilde{\lambda}_l$: $\tilde{J}_{k+l}(T) \to \tilde{J}_{(l,k)}(T)$. We remark that the sheaf Sol (R_k) of solutions of R_k is stable under the Lie bracket of vector fields. We say that R_k is *formally transitive* if π_0: $R_k \to J_0(T)$ is surjective. The differential equations $J_k(T;\varrho)$ and $J_k(V)$ considered in § 6 are Lie equations, and $J_k(T;\varrho)$ is formally transitive.

A differentiable sub-groupoid P_k of Q_k is a Lie equation (finite form) if it is a fibered submanifold of π: $Q_k \to X$. For $x \in X$, $I_k(x) \in P_k$ and $V_{I_k(x)}(P_k)$ determines a subspace $\tilde{R}_{k,x}$ of $\tilde{J}_k(T)_x$. The vector sub-bundle $R_k \subset J_k(T)$ such that $R_{k,x} = \nu(\tilde{R}_{k,x})$ is a Lie equation (infinitesimal form); we say that P_k is a finite form of R_k. For example, the sub-groupoids $Q_k(\varrho)$ and $Q_k(V)$ of Q_k are finite forms of $J_k(T;\varrho)$ and $J_k(V)$ respectively. We have $\tilde{R}_k \cdot F =$