DUALITY FOR CROSSED PRODUCTS AND THE STRUCTURE OF VON NEUMANN ALGEBRAS OF TYPE III

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1. Introduction

Undoubtedly, the principal problem in many field of mathematics is to understand and describe precisely the structure of the objects in question in terms of simpler (or more tactable) objects. After the fundamental classification of factors into those of type I, type II and type III by F. J. Murray and J. von Neumann, [25], the structure theory of von Neumann algebras has remained untractable in general form. It seems that the complete solution to this question is still out of sight. In the previous papers [44, 45], however, the author obtained a structure theorem for certain von Neumann algebras of type III in terms of a von Neumann algebra of type II_1 and an endomorphism of this algebra. Also,

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