

LACUNAS FOR HYPERBOLIC DIFFERENTIAL OPERATORS WITH CONSTANT COEFFICIENTS. II

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Introduction

Part I of this paper was published in 1970 (see the References [2]). Some of the results on lacunas of hyperbolic operators announced there depend on the fact, proved in all generality by Grothendieck [12] that the cohomology of a non-singular affine algebraic variety can be calculated from its rational differential forms with poles at infinity. Actually, a more precise result was needed putting a bound on the order of the poles. This result is now proved in Chapter I of this paper. It starts with a general account of the whole subject using only the basic results by Serre on algebraic coherent sheaves and Hironaka's resolution of singularities. For curves in the projective plane the results are final (§ 6). To obtain the desired bounds on the order of the poles, we use Grothendieck's generalizations of Serre's theorems to the framework of schemes.

In Chapter III we investigate the behaviour of the fundamental solution of a hyperbolic differential operator near the wave front surface. It starts with the observation that the fundamental solution has an analytic continuation across the wave front surface from a given side and at a given point y provided the Petrovsky homology class avoids the intersection of the corresponding hyperplane and the characteristic hypersurface. This condition is called the local Petrovsky condition. The global counterpart, appropriate when y is the origin, is simply that the Petrovsky class vanish. We use the local Petrovsky condition to verify, among other things that the fundamental solution has holomorphic extensions from both sides of a hyperplane part of the wave front surface provided one keeps away from the wave front surface (in the hyperplane) of the corresponding localization of the differential operator. There is also a formula for the jump of the fundamental solution across the hyperplane which shows that the singular support of the fundamental solution does *not* contain lacunas of the localization provided certain conditions of homogeneity are satisfied. This result connects the local Petrovsky condi-