# ENUMERATION UNDER TWO REPRESENTATIONS OF THE WREATH PRODUCT [1]

BY

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## 1. Introduction

Enumeration problems which can be solved by applying Pólya's Theorem [9] or Burnside's Lemma [1] always require a formula for $N(A)$, the number of orbits of group $A$, or a formula for its cycle index $Z(A)$. For example, Pólya [9] expressed the cycle index of the wreath product $A[B]$ of $A$ around $B$ in terms of the cycle indices $Z(A)$ and $Z(B)$. This result played a key role in the enumeration of $k$-colored graphs [13] and nonseparable graphs [14].

The exponentiation group $[B]^A$ of two permutation groups $A$ and $B$ was defined by Harary in [3]. It is abstractly isomorphic to the wreath product of $A$ around $B$. But while $A[B]$ has as its object set the cartesian product $X \times Y$ of the object sets of $A$ and $B$, $[B]^A$ acts on $Y^X$, the functions from $X$ into $Y$. Formulas for $Z([S_n]^{S_2})$ and $Z([S_2]^{S_n})$ were found by Harary [2] and Slepian [16] respectively. Harrison and High [6] have constructed an algorithm for finding $Z([B]^{S_n})$ and have used their results to enumerate Post functions. In this paper we verify an explicit general formula for $Z([B]^A)$ in terms of $Z(A)$ and $Z(B)$ for any $A$ and $B$. The result is easily obtained by substituting certain operators for the variables of $Z(A)$ and then letting them act on $Z(B)$. Several applications will then be sketched, including the enumeration of boolean functions, bicolored graphs, and Post functions.

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