THE STRUCTURE OF A VON NEUMANN ALGEBRA
WITH A HOMOGENEOUS PERIODIC STATE

(Dedicated to Professor M. Fukamiya on his 60th birthday)

BY

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Introduction

When F. J. Murray and J. von Neumann developed the theory of rings of operators
in the 1930's, they first classified all factors acting on separable Hilbert spaces into those
of type I, type II and type III. By showing that a factor of type I is isomorphic to the
algebra $\mathcal{L}(\mathfrak{H})$ of all bounded operators on some Hilbert space $\mathfrak{H}$, they proved that the
algebraic type of a factor of type I is completely determined by its dimension. Namely, the
factors of type I are classified into those of type I_n, $n = 1, 2, ..., \infty$. According to their
theory, we can not only classify the factors of type I, but also understand explicitly the
structure of a factor of type I. The situation is much worse for factors of types II and
III. Here we have not a complete classification. Furthermore, we had not been able to
construct many different factors until quite recently. To obtain infinitely many non-type

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