LOCALIZATION OF SHEAVES AND COUSIN COMPLEXES

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Introduction

One of the main difficulties in the theory of duality for coherent sheaves on schemes, or on analytic spaces, is the problem of joining locally defined objects of the derived category of the category of sheaves to a global object. Grothendieck presented a solution in the algebraic case (Hartshorne [4]) by showing that there is a category of complexes of sheaves, the injective Cousin complexes, which is equivalent to a subcategory of the derived category. It is then possible to join together locally defined objects of this subcategory.

The Cousin complexes are characterized in (Hartshorne [4]) by means of local cohomology. However, the procedure is not subject to immediate generalization, since it depends strongly on the special topological properties of the underlying space of a locally noetherian scheme. The purpose of this paper is to investigate the problem without restrictive hypotheses concerning the underlying space.

In section 1 we study localization in a category, in the sense of Gabriel [1], and its relation to local cohomology. For convenience we consider only categories of sheaves and localizing subcategories defined by subsets, though categories with injective envelopes may be treated in the same manner. In section 2 the results are extended to the category of complexes of sheaves. Also, Cousin complexes with respect to a filtration of the space are defined and some of their general properties are studied.

In section 3 we introduce a class of filtrations of the space, the admissible filtrations. The main result is Theorem 3.9, which shows that a subcategory of the category of Cousin complexes with respect to an admissible filtration is equivalent to a subcategory of the derived category.

In particular, when applied to locally noetherian schemes and filtrations defined by a codimension function (Hartshorne [4], V § 7), Theorem 3.9 implies that the category of all Cousin complexes is equivalent to a subcategory of the derived category.