THE COHOMOLOGY OF THE SPECTRUM OF A MEASURE ALGEBRA

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Let G be a locally compact abelian group, $M(G)$ the measure algebra on G, and Δ the spectrum or maximal ideal space of $M(G)$. It is common knowledge that, for non-discrete G, $M(G)$ is an extremely complicated Banach algebra with a very large spectrum which cannot be satisfactorily described. In fact, much of the research in the area has consisted of constructing measures in $M(G)$ which demonstrate that Δ fails to have a property one might have hoped for. For example, $M(G)$ is non-symmetric and, in fact, Δ contains infinite dimensional analytic structure (cf. [28], [18], [8], [19]); also, $M(G)$ has a proper Shilov boundary which is not the closure in Δ of the dual group of G (cf. [20], [12]). By contrast, one encouraging result on $M(G)$ is Cohen's Idempotent Theorem, which characterizes the idempotents in $M(G)$ (cf. [4]).

The purpose of this paper is to show that there is one sense in which $M(G)$ is surprisingly simple; specifically, the cohomology groups of its spectrum can be quite readily computed. In fact, to compute the cohomology groups of Δ one needs only to investigate the spectra of the algebras $L^1(G')$, where G' ranges over all l.c.a. groups which are continuously isomorphic to G. In degree zero this result is just Cohen's Idempotent Theorem. In degree one it leads to a characterization of those invertible measures in $M(G)$ which have logarithms in $M(G)$.

The class of algebras $M(G)$ is a subclass of the class of all convolution measure algebras. This larger class also contains the algebras $M(S)$ for S a locally compact topological semigroup and $L^1(G)$ for G a locally compact group. Our main results apply to any commutative, semi-simple convolution measure algebra.

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