BANACH SPACES WHOSE DUALS ARE L_1 SPACES AND THEIR REPRESENTING MATRICES

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1. Introduction

It is a matter of general agreement that the $L_p(\mu)$ spaces ($1 \leqslant p \leqslant \infty$ and μ a measure) and the $C(K)$ spaces (K compact Hausdorff) are among the most important Banach spaces. A central part of Banach space theory is devoted to the investigation of the special properties of these spaces and some closely related spaces. This part of Banach space theory is often called the theory of the classical Banach spaces. It is our feeling that in order to get a well rounded theory of the classical Banach spaces, in the framework of the isometric theory, it is worthwhile to take as the main objects of the investigation the class of Banach spaces X for which $X^* = L_p(\mu)$ for some $1 \leqslant p \leqslant \infty$ and some measure μ. Let us examine briefly the relation of this latter class of spaces to those mentioned in the first sentence. Since for $1 < p < \infty$ the $L_p(\mu)$ spaces are reflexive it is clear that $X^* = L_p(\mu)$ if and only if $X = L_q(\mu)$ $(p^{-1} + q^{-1} = 1)$. Grothendieck [6] proved the non obvious fact that if $X^* = L_\infty(\mu)$ then $X = L_1(\mu)$. Well-known results of F. Riesz and Kakutani show that if $X = C(K)$ then $X^* = L_1(\mu)$ for a suitable μ. There are, however, Banach spaces X which are not isometric to $C(K)$ spaces while their duals are $L_1(\mu)$ spaces. These are thus the only spaces which should be included in the geometric theory of the classical Banach spaces and which are not "classical" in the strict sense.

The most important geometric properties of the Banach spaces $C(K)$ are shared exactly by the class of all spaces whose duals are $L_1(\mu)$ spaces. Examples of such properties are the extension properties for compact operators which were studied in [14]. The $C(K)$ spaces are singled out from all the spaces whose duals are $L_1(\mu)$ mainly by the fact that they have natural additional structure as algebras or vector lattices. (There is, though, also a pure Banach space theoretic property which singles out the $C(K)$ spaces among