THE DISTRIBUTION OF THE VALUES OF ADDITIVE ARITHMETICAL FUNCTIONS

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Introduction

A real valued number theoretic function is said to be *additive* if for every pair of coprime positive integers a and b, the relation

$$f(ab) = f(a) + f(b)$$

is satisfied. Thus an additive function is determined by its values on the prime-powers. If, in addition, for each prime p

$$f(p) = f(p^2) = \dots,$$

then $f(m)$ is said to be *strongly additive*. In this paper we shall confine our attention to strongly additive functions.

The paper falls into three sections.

In the first section we consider those strongly additive functions $f(m)$ which, after a suitable translation, possess a limiting distribution. Theorems 1 and 2 provide a characterization of such functions, essentially in terms of their values on the primes.

A classic result of Erdös and Wintner states that an additive function $f(m)$ has a limiting distribution if and only if the two series

$$\sum_p \frac{f'(p)}{p} \qquad (*)$$

and

$$\sum_p \frac{(f'(p))^2}{p}$$

converge.[1] These two conditions are quite restrictive, however, so it is desirable to study

[1] See Notation.