A FREE BOUNDARY PROBLEM AND AN EXTENSION OF MUSKAT'S MODEL

BY

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1. Introduction

It is the purpose of this paper to derive and solve a mathematical model for the following physical problem. Suppose that in a homogeneous compressible porous medium one incompressible fluid is displacing another. The problem is to describe the motion of the fluids, in particular, the motion of the interface between the fluids, if the initial velocity distribution, or equivalently, the initial pressure distribution, of the fluids is given, together with appropriate boundary data.

We assume the flow to be in the horizontal x-direction, say, and neglect gravitational effects. We further assume the two fluids are immiscible so that for each time t there is a well defined interface between the fluids whose location is given by $x=\varrho(t)$. To the left of $\varrho(t)$ we denote the velocity of the fluid by $u(x, t)$ and its pressure by $p(x, t)$, and to the right we denote velocity and pressure by $v(x, t)$ and $q(x, t)$ respectively. The pressures and velocities are related by Darcy's law:

$$u(x, t) = -a\partial p(x, t)/\partial x, \quad v(x, t) = -b\partial q(x, t)/\partial x, \tag{1.1}$$

where a and b are positive quantities which depend on the physical properties of the fluid in question and of the porous medium and which we take to be constant. Since the fluids are incompressible, their densities are constant and the continuity equations take the form

$$\begin{cases} \partial\varphi/\partial t + \partial u/\partial x = 0 & x < \varrho(t), \\ \partial\varphi/\partial t + \partial v/\partial x = 0 & x > \varrho(t), \end{cases} \tag{1.2}$$

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