IDEAL THEORY AND LAPLACE TRANSFORMS FOR A CLASS OF MEASURE ALGEBRAS ON A GROUP

BY

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In this paper we introduce, and undertake the study of a class of Banach algebras associated with a locally compact group G. These algebras are related to the two-sided Laplace transform in the same way that the group algebra $L^1(G)$ and the measure algebra $M(G)$ are related to the Fourier transform. In the following paragraph, we indicate the nature of some of our final results by exposing them in the simplest nontrivial case.

If A is a compact convex subset of R^n let $\mathfrak{L}(A)$ denote the space of measurable functions on R^n for which

$$\|f\|_A = \int_{R^n} |f(x)|\,\varphi_A(x)\,dx < \infty, \quad \text{where} \quad \varphi_A(x) = \sup_{y \in A} e^{-x \cdot y}.$$

Note that for $f \in \mathfrak{L}(A)$, the Laplace transform

$$f^\wedge(z) = \int_{R^n} f(x)\,e^{-z \cdot x}\,dx$$

converges absolutely for $\operatorname{Re} z = (\operatorname{Re} z_1, ..., \operatorname{Re} z_n) \in A$. The following facts concerning $\mathfrak{L}(A)$ are special cases of results of this paper:

F1. (Lemma 2.2) $\mathfrak{L}(A)$ is a Banach algebra under the norm $\|\ \|_A$ and convolution multiplication;

F2. (Corollary to Theorem 6.1.) The maximal ideal space of $\mathfrak{L}(A)$ can be identified with $\{z \in \mathbf{C}^n : \operatorname{Re} z \in A\}$, and the Gelfand transform of $f \in \mathfrak{L}(A)$ can be identified with the Laplace transform $f^\wedge$ restricted to $\{z \in \mathbf{C}^n : \operatorname{Re} z \in A\}$;

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