COMMUTATORS AND SYSTEMS OF SINGULAR INTEGRAL EQUATIONS. I[1]

BY

JOEL DAVID PINCUS

Brookhaven National Laboratory, Upton, N.Y. and Courant Institute of Matematical Sciences, New York University, New York, N.Y., U.S.A.[2] [3]

Introduction

In this paper we study certain self-adjoint singular integral operators with matrix coefficients acting on a multi-component Hilbert space H; namely,

$$Lx(\lambda) = A(\lambda)\,x(\lambda) + \frac{1}{\pi i}\mathbf{P}\int_a^b \frac{k^*(\lambda)\,k(\mu)}{\mu-\lambda}\,x(\mu)\,d\mu,$$

where

$$A(\lambda) = \begin{pmatrix} A_{11}(\lambda) & \dots & A_{1n}(\lambda) \\ A_{21}(\lambda) & \dots & A_{2n}(\lambda) \\ \vdots & \vdots & \vdots \\ A_{n1}(\lambda) & \dots & A_{nn}(\lambda) \end{pmatrix},$$

$$k(\lambda) = \begin{pmatrix} k_{11}(\lambda) & \dots & k_{1n}(\lambda) \\ k_{21}(\lambda) & \dots & k_{2n}(\lambda) \\ \vdots & \vdots & \vdots \\ k_{n1}(\lambda) & \dots & k_{nn}(\lambda) \end{pmatrix},$$

where the matrices above have elements which are complex-valued functions of λ, and for almost all λ, $A(\lambda)$ is a bounded Hermitian operator on the Hilbert space H which consists of vectors $x(\lambda) = \{x_1(\lambda), \dots, x_n(\lambda)\}$ with measurable components such that

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(3) Present address: State University of New York at Stony Brook, Stony Brook, L. I., N.Y. U.S.A.