GROUPS OF ORDER 1

SOME PROPERTIES OF PRESENTATIONS

BY

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Section 1

A presentation of deficiency zero (on n symbols and n defining relations) of a group G may define the trivial group, $G=1$.

The present work is a contribution to the decision problem: when does the presentation

$$P: (a_1, \ldots, a_n; r_1(a), \ldots, r_n(a))$$

of G give the trivial group?

It can be decided at once whether the r_i freely generate the free group $F_n = F(a)$ (see [12]). The question is how to reduce P to this case if $G=1$.

The next simplest case is that all but one of the r_i form a set of associated generators (one that can be completed to a free generating set of F_n) [8]. The simple fact that the consequence of such a set $(r_1, \ldots, r_{n-1})$ contains the commutator subgroup F' of F_n motivates the introduction of what I will call root-extraction. For example if $(a_1, a_2; a_1, r_2)=1$ then there is a word s_2 such that a_1 and s_2 generate $F_2 = F(a)$ and $r_2 \equiv a_2$ modulo a_1 and $r_2 \equiv s_2$ modulo s_2. (See Sections 4 and 5.)

The introduction of Nielsen transformations (automorphisms of free groups) combined with conjugations—I will call these Q-transformations—hardly needs motivating in this context. Root-extraction on t-tuples $r = (r_1(a), \ldots, r_t(a))$ in $F_n = F(a)$ will consist of replacing a proper subset of r by another set without changing normal closure and deficiency of presentation.

For n-tuples r for which the presentation P above is that of the trivial group, the fol-

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