NONLINEAR EIGENVALUE PROBLEMS

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Introduction

Let $A, B_0, ..., B_h$ be compact linear operators on a Hilbert space H and let p and q be integers, $p \geqslant 0$ and $q > 0$. Consider the operator

$$C(\lambda) = \lambda^q I - A - \sum_{k=0}^{h} \lambda^{p+k} B_k, \tag{0.1}$$

where I is the identity operator and λ is any complex number. If for some λ there is a non-zero element u such that $C(\lambda)u=0$ then we say that λ is an *eigenvalue* and u an *eigenvector* of $C(\lambda)$. In case $C(\lambda)=\lambda I - A$ and A is self-adjoint, a classical result asserts that the eigenvectors of $C(\lambda)$ (or of A) are complete. There is also a completeness theorem in case A is not

(1) The first author is partially supported by National Science Foundation NSF GP-5558. The second author is partially supported by National Science Foundation NSF GP-6632.