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Algebraic K-theory of spaces, with bounded control

by

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This paper is concerned with a boundedly controlled version of the algebraic K-theory of spaces functor $X \mapsto A(X)$. The word *boundedly controlled* refers to the following situation. Every object is equipped with a reference map to a metric space. In particular it makes sense to talk about *boundedness* of maps, homotopies etc.

Controlled algebraic K-theory should be related to *bounded* stable concordance theory in the same way as algebraic K-theory of spaces is related to (ordinary) stable concordance theory.

Parts of such a theory have been studied variously.

As a first example consider h-cobordisms W with a reference map $p: W \to B$ to a metric space B. One may ask when W has a *bounded product structure*. In [AH] Anderson and Hsiang have been studying such cobordisms in the special case where the lower boundary of W is of the form $M \times \mathbf{R}^k$, where M is a compact manifold and the metric space is $\mathbf{R}^k$. The answer to this question is provided by the *bounded s-cobordism theorem*. It turns out that there is a naturally defined algebraic K-theory invariant whose vanishing guarantees the existence of a bounded product structure on W. The group in which these invariants live is called the *controlled Whitehead group*.

As another example, this time on the K_0-level, we mention the *controlled finiteness obstruction*, which has been treated for example in [C]. The problem here is to decide when a space which is finitely dominated in the bounded sense is actually boundedly homotopy equivalent to a locally finite space.

In [PW1, PW2] Pedersen and Weibel have been studying a version of controlled algebraic K-theory of rings. They define the category $\mathfrak{C}_n(F_R)$ of locally finite families of free R-modules parametrized by $\mathbf{Z}^n$. They show that its K-theory is in fact an n-fold (non-connective) de-looping of the K-theory of the ring R. In [PW2] this result is generalized: To any metric space B there is associated a category $\mathfrak{C}_B(F_R)$ of locally finite families of free R-modules parametrized by the metric space B. Now assume that the metric space arises in the following way. Let X be a finite PL-subcomplex of $\mathbf{R}^\infty$,