ON THE NUMBER OF DIVISORS OF QUADRATIC POLYNOMIALS

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1. Introduction

The problem of determining the asymptotic behaviour, as $x \to \infty$, of the divisor sum

$$S(x) = \sum_{n \leqslant x} d(n^2 + a),$$

where $d(\mu)$ denotes the number of (positive) divisors of μ, has been mentioned by a number of writers [1], [2], [5], [8]. When we consider this problem it is not difficult to see that the case where $-a$ is a perfect square k^2, say, is exceptional, since then $n^2 + a$ can be factorized as $(n-k)(n+k)$. In this case the sum is almost identical with the sum

$$\sum_{n \leqslant x} d(n)\, d(n+2k),$$

which has been considered by Ingham [7]; in fact a slight adaptation of Ingham's method shews here that

$$S(x) = A_1(a)\, x \log^2 x + O(x \log x) \qquad (a = -k^2).$$

We shall not, therefore, refer to this case again. In the case when $-a$ is not a perfect square for some considerable time it has been commonly realized (see, for example, the remarks by Bellman [1] and the author [5]) that it is possible to deduce an asymptotic formula

$$S(x) = A_2(a)\, x \log x + O(x)$$

by a familiar elementary method; a proof of such a formula (with a less precise error term) has recently been supplied by Scourfield [8].