EMBEDDING THEOREMS FOR LOCAL ANALYTIC GROUPS

BY

S. ŚWIERCZKOWSKI

University of Sussex, Brighton, England

1. Results and fundamental concepts

Results

A Banach space X in which there is defined a continuous Lie multiplication $[x,y]$ will be called a normed Lie algebra. One can assign to every normed Lie algebra X a local group consisting of a sufficiently small neighbourhood of 0 in X in which the multiplication xy is given by the Campbell–Hausdorff–Schur formula

$$xy = x + y + \tfrac{1}{2}[xy] + \tfrac{1}{12}[y[yx]] + \tfrac{1}{12}[x[yx]] + \dots$$

(Birkhoff [3], Cartier [5] and Dynkin [10]). Let us denote this local group by $L(X)$. If X is finite dimensional, then $L(X)$ is of Lie type and therefore it is always locally embeddable in a group (Ado [1], Cartan [4], Pontrjagin [17]). We shall say that a normed Lie algebra X is an E-algebra if $L(X)$ is locally embeddable in a group. Since it has been discovered recently that not all normed Lie algebras are E-algebras (van Est and Korthagen [11]), it is natural to ask which of them are. In this direction we prove

THEOREM 1. *If X is a normed Lie algebra, $Y \subset X$ is a closed ideal and*

a) *the Lie algebra X/Y is abelian,*

b) *Y is an E-algebra,*

then X is an E-algebra.

We shall use this theorem in order to prove that an algebra X which is soluble, or soluble in a generalised sense is always an E-algebra. More precisely, let us say that the normed Lie algebra X is *lower soluble* if there exists an ordinal number α and an ascending sequence

$$\{0\} = X_0 \subset X_1 \subset X_2 \subset \dots X_\beta \subset X_{\beta+1} \subset \dots \subset X_\alpha = X$$

of closed subalgebras of X such that

14 – 652933 *Acta mathematica* 114. Imprimé le 19 octobre 1965.