ALGEBRAS OF BOUNDED ANALYTIC FUNCTIONS ON RIEMANN SURFACES

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Let (W, Γ) be a bordered Riemann surface. We say that (W, Γ) satisfies the *AB*-maximum principle if each bounded analytic function on $W \cup \Gamma$ assumes its maximum on Γ. If W has genus zero, then W is conformally equivalent to a plane domain whose boundary consists of the image of Γ and a totally disconnected perfect set E which has the property that it is "removable" for every bounded analytic function, i.e. every function bounded and analytic in $U \sim E$ for some neighborhood U of E can be extended to be analytic in U. A similar situation occurs when W has finite genus, but when W has infinite genus we cannot represent it as the complement of a set on a compact surface, and so the question arises of whether we can have some generalization of this notion of "removability".

Myrberg [4] and Selberg [6] have shown that for certain twosheeted covering surfaces of the disc each bounded analytic function on the surface is obtained by lifting a bounded analytic function from the disc to the surface, and Heins [2] has shown that, if W has a single end and that a parabolic end, then there is a mapping φ of that end into a disc so that every bounded analytic function on the end is of the form $f \circ \varphi$ where f is analytic on the disc. In the present paper we generalize these results by establishing Theorem 3 which states that, if (W, Γ) satisfies the *AB*-maximum principle, then there is an analytic mapping φ of $W \cup \Gamma$ into a compact Riemann surface such that each bounded analytic function on $W \cup \Gamma$ is of the form $f \circ \varphi$ where f is an analytic function in a neighborhood of the closure of $\varphi[W \cup \Gamma]$.

The proof of this theorem relies on the application of techniques from the theory of function algebras to the algebra of bounded analytic functions on $W \cup \Gamma$. We begin in Section 1 by showing that, if we have any algebra of analytic functions on a Riemann

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