ON DEFORMATIONS OF DISCONTINUOUS GROUPS

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Let D be a product of irreducible bounded symmetric domains in the complex number space and let Γ be a properly discontinuous group on D with the property that $\operatorname{vol}(D/\Gamma)$ is finite.

If one excludes that D has any components of complex dimension 1, it is generally suspected (cf. [15]) that any such group must be commensurable to an arithmetic group.

In particular, if this is the case, there will be no other families of discontinuous groups containing Γ except those obtained by operating on Γ by a family of inner automorphisms of the Lie group $G = \operatorname{Aut}(D)$

Under the more stringent assumption that D/Γ be compact, this has proved to be the case in [15] and in [7] as a consequence of a more general rigidity theorem. That result has been extended by A. Weil [21] to the case of all "reasonable" semisimple Lie groups (i.e., a semisimple Lie group without compact components whose Lie algebra has no simple factor of dimension 3).

In the case where D/Γ is not compact it has been stated by A. Selberg (in a conversation with one of the authors at the international congress in Stockholm) that at least the following should be true:

Let us suppose that Γ_1 and Γ_2 are two properly discontinuous groups on D and suppose that (a) Γ_1 is an arithmetic group, (b) there exist fundamental domains F_1, F_2 for Γ_1, Γ_2 respectively, such that, outside of a compact set $K \subset D$, $F_1 - F_1 \cap K = F_2 - F_2 \cap K$. Then Γ_2 must be commensurable with Γ_1.

Again, if this is the case, there will be only trivial families of discontinuous

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