A MAXIMAL THEOREM FOR SUBADDITIVE FUNCTIONS

BY

R. P. GOSSELIN

University of Connecticut (¹)

Introduction

The theory of subadditive functions is sufficiently well developed to suggest that it may be a very useful tool of analysis. The present paper, in which we first prove a maximal theorem for subadditive functions and then apply it to a rather wide class of problems, is offered as further evidence of this point of view.

Our maximal theorem does not seem to be included in the category of maximal ergodic theorems. It does have some points of contact with that of Hardy and Littlewood, but the situation is roughly that our theorem gives more precise information about a smaller class of functions. We first consider some variations of the definition of subadditivity of real-valued functions defined over E_n, n-dimensional Euclidean space. For the maximal theorem itself, a kind of evenness of the functions involved is assumed. We then construct the maximal function corresponding to each properly chosen subadditive function; and the maximal theorem, which is a statement about the comparability of some integral norms involving the original function and its corresponding maximal function is given. In the second theorem, some limitations on the maximal theorem are noted. In the next section, applications are presented, first for some well-known subadditive functions to which the maximal theorem applies directly. A minor variation of the theorem is then applied to some integral transforms. Finally, we obtain a kind of local maximal theorem in a result which is related to the differentiability of integrals. Modifications of the original argument are more serious for this result, and we make use of the maximal theorem of Hardy and Littlewood here. In the last section, sums whose terms involve subadditive functions are introduced. The main result of this section is a statement about the