LOCAL BEHAVIOR OF SOLUTIONS OF QUASI-LINEAR EQUATIONS

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This paper deals with the local behavior of solutions of quasi-linear partial differential equations of second order in $n \geqslant 2$ independent variables. We shall be concerned specifically with the *a priori* majorization of solutions, the nature of removable singularities, and the behavior of a positive solution in the neighborhood of an isolated singularity. Corresponding results are for the most part well known for the case of the Laplace equation; roughly speaking, our work constitutes an extension of these results to a wide class of non-linear equations.

Throughout the paper we are concerned with real quasi-linear equations of the general form

$$\operatorname{div} \mathcal{A}(x, u, u_x) = \mathcal{B}(x, u, u_x). \tag{1}$$

Here $\mathcal{A}$ is a given vector function of the variables x, u, u_x, $\mathcal{B}$ is a given scalar function of the same variables, and $u_x = (\partial u/\partial x_1, \ldots, \partial u/\partial x_n)$ denotes the gradient of the dependent variable $u = u(x)$, where $x = (x_1, \ldots, x_n)$. The structure of (1) is determined by the functions $\mathcal{A}(x, u, p)$ and $\mathcal{B}(x, u, p)$. We assume that they are defined for all points x in some connected open set (domain) Ω of the Euclidean number space E^n, and for all values of u and p. Furthermore, they are to satisfy inequalities of the form

$$\left.\begin{aligned} |\mathcal{A}| &\leqslant a|p|^{\alpha-1} + b|u|^{\alpha-1} + e, \\ |\mathcal{B}| &\leqslant c|p|^{\alpha-1} + d|u|^{\alpha-1} + f, \\ p \cdot \mathcal{A} &\geqslant a^{-1}|p|^{\alpha} - d|u|^{\alpha} - g. \end{aligned}\right\} \tag{2}$$

Here $\alpha > 1$ is a fixed exponent, a is a positive constant, and the coefficients b through g are measurable functions of x, contained in certain definite Lebesgue classes over Ω (see Chapter I).