ISOTROPIC INFINITELY DIVISIBLE MEASURES ON SYMMETRIC SPACES

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§ 1. Introduction

It has been clear for some time that a natural domain for questions of harmonic analysis is the class of symmetric spaces G/K where G is an appropriate Lie group and K an appropriate subgroup [10], [14]. (2)

Now, functions, measures etc. on G/K may be viewed as corresponding objects on G which are invariant under the right action of K, and a convolution operation may be defined for them via the group structure of G.

In this paper we study the representation of probability measures on G/K which are isotropic in the sense that they are, as measures on G, also invariant under the left action of K, and which are infinitely divisible in the sense of the convolution mentioned above. The representation is carried out via the abstract Fourier–Stieltjes transform [13], and the main result is Theorem 6.2 which is analogous to the celebrated Lévy–Khinchine formula for the characteristic function of an infinitely divisible probability measure on the real line.

Certain other results of probabilistic significance are also obtained. The principal one is Theorem 7.2 which is the analogue of a classical theorem of Khinchine [18].

The organization of this paper is as follows. § 2, § 3 are devoted to terminology and recapitulation of known results which will be used often in the sequel. § 4–7 are devoted to the proofs of our theorems in the case when G/K is a symmetric space of the non-compact type. § 8 then indicates briefly the modifications to be made when G/K is of the compact type.

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(2) Numbers in square brackets refer to the Bibliography at the end of this paper.