FREDHOLM EIGENVALUES AND QUASICONFORMAL MAPPING

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1. Introduction

Let $\tilde{D}$ be a region of connectivity n in the z-plane which contains the point of infinity and whose boundary C consists of n smooth Jordan curves $C_1, C_2, \ldots, C_n$. Each curve C_j is the boundary of a bounded simply connected region D_j and we write $D = \bigcup_{j=1}^{n} D_j$. The Neumann–Poincaré integral equation is

$$f(s) = \lambda \int_C K(s, t) f(t)\, dt, \tag{1}$$

where s and t represent the arc length parameter on C, $z(s)$ is a parametric representation of C in terms of its arc length, $\partial/\partial n_t$ represents differentiation in the direction of the inward normal at $z(t)$, and

$$K(s, t) = \frac{\partial}{\partial n_t} \log \frac{1}{|z(s) - z(t)|}. \tag{2}$$

This integral equation plays an important role in potential theory and conformal mapping. It can be solved by iteration and the Neumann–Liouville series so obtained converges like a geometric series whose ratio is $1/|\lambda|$ where λ is the lowest eigenvalue of (1) whose absolute value is greater than one. The eigenvalues of (1) are known as the Fredholm eigenvalues of C. They are all real, satisfy $|\lambda| \geqslant 1$, and those for which $|\lambda| > 1$ lie symmetrically about the origin. Those of modulus one are referred to as the trivial eigenvalues. In order to have an estimate for the rate of convergence of the Nenmann–Liouville series, it has been an important problem to estimate from below the lowest non-trivial positive Fredholm eigenvalue, which will be denoted by λ in what follows.

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