LIPSCHITZ APPROXIMATIONS TO SUMMABLE FUNCTIONS

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1. Introduction

In [4] I considered a continuous non-negative function ϕ on R^n with certain special properties and I defined for each Lipschitz function f on the closed unit cube Q of R^n,

$$\Psi(f) = \int_Q \phi(\text{grad } f)\, dx.$$

This non-negative functional Ψ was shown to be lower semi-continuous on the set of Lipschitz functions with the $\mathcal{L}_1$ topology and hence could be extended to a non-negative lower semi-continuous functional on the summable functions. The main result of [4] was the following.

If f is continuous on Q and such that $\Psi(f)$ is finite, and if $\varepsilon > 0$, then there exists a Lipschitz function g on Q such that the set

$$\{x;\, x \in Q \text{ and } f(x) \neq g(x)\}$$

has measure less than ε and $\Psi(g) < \Psi(f) + \varepsilon$. This problem arose from a conjecture of C. Goffman concerning the approximation of non-parametric surfaces with finite area, by Lipschitz surfaces. See [2] and [3].

In the present paper this theorem is proved without the continuity restriction, i.e., it is shown that any function f, summable on Q and with $\Psi(f) < \infty$, can be approximated by a Lipschitz function in the manner described above. Also, in the new theorem, a more general ϕ is taken, hence a more general functional Ψ.

Throughout the present paper ϕ denotes a non-negative continuous, real-valued function on R^n with the following properties:

(i) $\phi(\xi) \geqslant \phi(\xi')$, when $|\xi_1| \geqslant |\xi_1'|, \ldots, |\xi_n| \geqslant |\xi_n'|$;