REMARK ON A PROBLEM OF LUSIN

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1. In 1915, (see [2] for an edition with added commentary), Lusin asked whether, for every measurable function f on $[0, 2\pi]$, finite or infinite, there is a trigonometric series, with coefficients converging to zero, which converges almost everywhere to f.

The problem was solved in the affirmative by Menchoff, [3], [4] (also, see [1]), for the case where f is finite almost everywhere. Bari, ([2], p. 527), also solved the problem for the finite case, with Haar functions instead of trigonometric functions; an interesting but easier bit of mathematics.

By substituting convergence in measure for almost everywhere convergence, Menchoff, [5], then answered Lusin's question. He showed that for every measurable f on $[0, 2\pi]$, finite or infinite, there is a trigonometric series, with coefficients converging to zero, which converges in measure to f. This work of Menchoff is difficult to understand. Fortunately, Talalyan has given a brilliant and lucid treatment of this problem, summarized in [7], where he proves Menchoff's theorem for every normal Schauder basis in $L_p[a, b]$, $p > 1$.

The original Lusin problem remains unanswered, not only for the trigonometric functions but for any Schauder basis in any L_p, $p > 1$. It is not even known whether any such series converges almost everywhere to $+\infty$; in particular, this is not known for the Haar functions.

Schauder, [6], originally introduced the idea of basis for the space $C[0, 1]$ as well as for the L_p spaces. It is natural to ask whether Lusin's problem has an affirmative answer using this system of functions. It is our purpose here to show that it does. The problem for this case is, of course, of a much lower order of difficulty than for the original trigonometric functions, or even for the Haar functions. Nevertheless, it turns out to be of technical interest in its own right.

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